Cosmological components in the presence of perfect fluid for Robertson-Walker cosmological model in general relativity

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Abstract

Different types of cosmological constants are considered for analyzing perfect fluid. Different gravitational variable and the cosmological constant are considered within sight of perfect fluid for the Robertson-Walker universe by utilizing Einstein's field equations. The cosmological term to be proportional to R^{-m} (R is a scale factor and m is a constant). Varieties of cosmological components are presented for the presence of perfect fluid in the equation of state.

Key Words: Perfect fluid, Robertson-Walker metric, Cosmology, Gravitational variable, Equation of state.

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1. Introduction

At the present time, it is a challenging issue to explain the small positive value of the cosmological constant for the energy density of the vacuum. The observed fluid variable value 7 x 10^{-30} cm^{-3} is over in various 120 orders sample that magnitude is smaller than the Planck density (10^{93} g cm^{-3}). This value of the universe is risen up out of the big bang and its value is set at that time. These particles of the universe are decaying every day. A number of models with different decaying laws for the variation of cosmological terms were investigated in previous. The variation in the fluid is joined by a variation of G. Several authors investigated Friedman-Roberson-walker (FRW) models and Bianchi models using this approach [1, 3, 16, 26, 30, 32, 35, 38, 39]. In previous research have considered that the cosmological term is proportional to the Hubble parameter in the FRW model and the Bianchi type-I model with variable G and Λ[10]. Classification of the FRW universe with a cosmological constant and a perfect fluid in the equation of state have been examined and proposed the idea of a variable gravitational constant G in the framework of general relativity[13, 14]. Previous researcher working in the structure of general relativity, proposed an adjustment connecting the variation of G with that of Λ[18]. Several researchers proposed a modification approach that allows us to use Einstein's field equations' form, which is unchanged in cosmological scenarios with a time varying Λ. Recently, we have considered [35, 36, 37] whether the cosmological term is corresponding to the Hubble parameter in the Bianchi type-I model and FRW model with varying G and Λ. In this paper, we think about homogeneous R-W space-time with variable G and Λ containing matter as a perfect fluid with a scale factor. Here we represent the relationship between growth rate and time in the presence of perfect fluid for the matter dominated universe and the radiation dominated universe. We obtain solution from the field equations assumption that the cosmological term is proportional to $\mathfrak{R}^{-m}$ (where $\mathfrak{R}$ is a scale factor and m is constant). This paper is organized as follows. Basic equations of the models are given in section 2; and their solution is in section 3; we examine the model in section 4 and conclude our outcomes in section 5.

2. The model and field equations

Robertson-Walker (R-W) model is the standard model of the universe for homogeneous and isotropic condition. The spatially homogeneous and isotropic Robertson-Walker (R-W) model is given by

$$ds^2 = dt^2 - \mathrm{R}^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\emptyset^2)\right], \qquad (1)$$

where the scale factor is R (t) and the curvature parameter k for an open, flat or closed universe is $k = -1, 0$ or $+1$ respectively.

We assume that the energy-momentum tensor of a perfect fluid which is represented for cosmic matter,

$$T_{\eta\mu=(\rho+p)u_\mu u_\mu - p g_{\eta\mu,}} \qquad (2)$$

where ρ is the energy density of cosmic matter, p is its pressure, and u_η is the four velocity vector such that $u_\eta u^\eta = 1$. We consider the equation of state

$$p = (\lambda - 1)\rho, 1 \leq \lambda \leq 2. \qquad (3)$$

The Einstein field equations with variable G and Λ are given by Weinberg (1972)

$$\mathrm{R}_{\eta\mu} - \frac{1}{2}\mathrm{R}g_{\eta\mu} = 8\pi G(t)T_{\eta\mu} + \Lambda\,(t)g_{\eta\mu.} \qquad (4)$$

In a commoving system of coordinates, using the metric equation (1) and energy – momentum tensor equation (2) the field equation (4) yields,

$$-\frac{2\ddot{\mathrm{R}}}{\mathrm{R}} - \frac{\dot{\mathrm{R}}^2}{\mathrm{R}^2} - \frac{k}{\mathrm{R}^2} = 8\pi G p - \Lambda\,, \qquad (5)$$

$$\frac{3\dot{\mathrm{R}}^2}{\mathrm{R}^2} - \frac{3k}{\mathrm{R}^2} = 8\pi G\rho + \Lambda\,. \qquad (6)$$

Since the divergence of Einstein tensor is vanishing, we have

$$8\pi G\left\{\dot{\rho} + 3(\rho + p)\frac{\dot{\mathrm{R}}}{\mathrm{R}}\right\} + 8\pi\rho\dot{G} + \dot{\lambda} = 0\,. \qquad (7)$$

The law of energy conservation equation $T^{j}_{i,j} = 0$ yields.

$$\dot{\rho} + 3(\rho + p)\frac{\dot{\mathrm{R}}}{\mathrm{R}} = 0\,. \tag{8}$$

So that equation (7) reduces to

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0. \tag{9}$$

The expression of equation (9) implies that Λ is a constant whenever G is constant.

Putting the values of p from equation (3) in equation (8) and then integrating, we get $\rho = \frac{k_1}{\mathrm{R}^{3\lambda}}$, (10)

here k_1 is a constant of integration. Equations (5) and (6) can be modified as far as Hubble parameter H and deceleration parameter q for zero curvature (k=0) as,

$$H^2(2q-1) = 8\pi G p - \Lambda\,, \tag{11}$$

$$3H^2 = 8\pi G\rho + \Lambda\,, \tag{12}$$

where $q = -1 - \frac{\dot{H}}{H^2} = -\frac{\mathrm{R}\ddot{\mathrm{R}}}{\dot{\mathrm{R}}^2}$ and expansion scalar $\theta = 3H = \frac{3\dot{\mathrm{R}}}{\mathrm{R}}$. The critical density ρ_c, vacuum density ρ_v and density parameter Ω was defined[23] as

$$\rho_c = \frac{3H^2}{8\pi G}\,, \tag{13}$$

$$\rho_v = \frac{\Lambda}{8\pi G}\,, \tag{14}$$

And $$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2}\,, \tag{15}$$

3. Solution of the field equations

The system of equation (3), (5), (6) and (8) supply only four equations in five unknown's $(R, \rho, p, G\ and\ \Lambda)$. One additional equation is expected to explain the framework totally. The phenomenological Λ decay scenarios have been considered by a number of authors[11, 15] consider $\Lambda \propto \mathrm{ą}^{-2}$(ą is the scale factor of the Robertson–Walker metric). Consider $\Lambda \propto \mathrm{ą}^{-3}$ while $\Lambda \propto \mathrm{ą}^{-m}$ (ą is the

scale factor and m is a constant) was considered by several researcher[9, 20, 22, 24, 28, 29].

Thus we take the decaying vacuum energy density

$$\Lambda = \frac{ą}{Ř^m}, \tag{16}$$

where ą is constant. From equations (9), (10) and (16) one finds that

$$G = \frac{ąŘ^{3\lambda-m}}{8k_1(3\lambda-m)} \equiv G_0 Ř^{3\lambda-m}. \tag{17}$$

There equations (5) and (6) can be written in the form of perfect fluid

$p_{eff} = (\lambda_{eff} - 1)p_{eff}$, With

$$p_{eff} = \left(k_1 G_0 + \frac{ą}{8\pi}\right)\frac{1}{G_n Ř^m}, \tag{18}$$

where G_n is the Newtonian Gravitational constant and λ_{eff} can be given explicitly regarding different constants .Using Equations (3), (11), (12) and (16) we get a differential equation

$$2\dot{H} + 3\lambda H^2 - \frac{ą\lambda}{Ř^m} = 0 \tag{19}$$

On integrating equation (19), the scale factor obtain that

$$Ř = \left(\frac{m}{2}\sqrt{\frac{ą\lambda}{3\lambda-m}}\,t + t_0\right)^{2/m}, \tag{20}$$

where t_0 is a constant of integration. The integration constant is related to the choice of origin of time.

Since $1 \leq \lambda \leq 2$ now we analyze the scenarios of cosmological components for $\lambda = \frac{3}{2}$

3.1 Solution of cosmological components for $\boldsymbol{\lambda = \frac{3}{2}}$

For $\lambda = \frac{3}{2}$ from equation (20), we obtain the scale factor

$$\text{Ŗ} = \left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{2/m} \tag{21}$$

In this case, the spatial volume V, matter density ρ , pressure p, gravitational constant G, and cosmological constant Λ and others cosmological components are given by the following:

solution for $\lambda = \frac{3}{2}$ at m=1
$$\text{Ŗ} = \left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{2/m}$$ $$= \left(\frac{1}{2}\sqrt{\frac{3ą}{7}}t + t_0\right)^2 \quad (22)$$
$$V = \left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{6/m}$$ $$= \left[\frac{1}{2}\sqrt{\left(\frac{3ą}{7}\right)}\ t + t_0\right]^6 \quad (23)$$
$$\rho = \frac{k_1}{\left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{9/m}}$$ $$= \frac{k_1}{\left(\frac{1}{2}\sqrt{\frac{3ą}{7}}t+t_0\right)^9} \quad (24)$$

$$p = \frac{1}{2}\frac{k_1}{\left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{9/m}}$$

$$\frac{1}{2}\frac{k_1}{\left(\frac{1}{2}\sqrt{\frac{3ą}{7}}t+t_0\right)^{9}} \tag{25}$$

$$G = G_0\left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{\frac{9}{2}-m}$$

$$= G_0\left(\frac{1}{2}\sqrt{\frac{3ą}{7}}t + t_0\right)^{\frac{7}{2}} \tag{26}$$

$$\Lambda = \frac{ą}{\left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{2}}$$

$$= \frac{ą}{\left[\frac{1}{2}\sqrt{\left(\frac{3ą}{7}\right)}t + t_0\right]^{2}} \tag{27}$$

$$\theta = \frac{3\sqrt{3ą}}{\sqrt{9-2m}}\left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{-1}$$

$$= \frac{3\sqrt{3ą}}{\sqrt{7}}\left[\frac{1}{2}\sqrt{\left(\frac{3ą}{7}\right)}t + t_0\right]^{-1} \tag{28}$$

$$\sigma = \frac{c_1}{\sqrt{3}}\left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{6/m}$$

$$= \frac{c_1}{\sqrt{3}}\left(\frac{1}{2}\sqrt{\frac{3ą}{7}}t + t_0\right)^{6} \tag{29}$$

The vacuum energy density ρ_v and critical density ρ_c are given by $\rho_v = \frac{ą}{8\pi G_0}\left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{9/2}$, $\rho_c = \frac{9ą}{8\pi G_0(9-2m)}\left(\frac{m}{2}\sqrt{\frac{3ą}{9-2m}}t + t_0\right)^{-\frac{13}{2}+m}$, The density parameter is given by $\Omega = \mathrm{m} - 2$	The vacuum energy density ρ_v and critical density ρ_c are given by $\rho_v = \frac{ą}{8\pi G_0}\left(\frac{1}{2}\sqrt{\frac{3ą}{7}}t + t_0\right)^{9/2}$, (30) $\rho_c = \frac{9ą}{56\pi G_0}\left(\frac{1}{2}\sqrt{\frac{3ą}{7}}t + t_0\right)^{-\frac{11}{2}}$, (31) The density parameter is given by $\Omega = -1$ (32)
The deceleration parameter q for the model is $q = \frac{m}{2} - 1$.	The deceleration parameter q for the model is $q = -\frac{1}{2}$ (33)

We examine that for $0 < m < 4$ the spatial volume V is zero att $= \mathrm{t}^*$, where $t^* = -2t_0/\left(\sqrt{\frac{3ą}{7}}\right)$ and expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume at $\mathrm{t} = t^*$ with an infinite rate of expansion. The scale factor also vanishes at $\mathrm{t} = t^*$, hence the space-time exhibits point type singularity in the initial epoch. The energy density and hence the space-time exhibits point type singularity in the initial epoch. The energy density and shear scalar move away at the initial singularity. The scale factor and spatial volume increases however expansion scalar decreases as t increases. Thus, the rate of extension backs off with increasing time. Moreover $\rho, \sigma, \rho_v, \rho_c$ and Λ decrease as t increases. As $\mathrm{t} \to \infty$, the scale factor and volume become infinite, $\rho, \sigma, \rho_v, \rho_c$ and Λ tend to zero. Therefore, the model would basically give a vacant universe for substantial time t. Gravitational constant $G(t)$ is zero at $t = t^*$ and as t increases, $G(t)$ also increases. Therefore, the universe starts with decelerating extension changes and the development changes from a decelerating stage to an accelerating one. Here we represent

cosmological behavior for m=1which satisfies the cosmological scenario is in agreement with SNe Ia astronomical observations [17, 27, 34] and it presents a unified description of the evolution of the universe.

4. Discussion

In this work, we have presented a variety of solutions to a spatially homogeneous and isotropic R-W line element with variable gravitational constant G(t) and cosmological constant Λ(t). By using law of variation of scale factor with a variable cosmological term, i.e. a cosmological term that scales as $\Lambda \propto \mathrm{R}^{-m}$ (where R is a scale factor) the field equations have been solved exactly. In sections 3.1 exact $R - W$ models have been obtained for m=1. Some important cosmological parameters were obtained for the model and the physical behaviors of the model were discussed in detail in sections 3.1 by mathematical expression. In the cases, the models represent a shearing, non-rotating and expanding model of the universe with a big-bang start which approaches isotropy at late times. It is interesting that the proposed variety law gives an elective way to deal with getting exact solutions of Einstein's field conditions. It shows a unified description of the development of the universe which begins with a decelerating extension and grows with acceleration at late times. Recent observational information [16, 17] strongly suggests this acceleration occurs. Also, gravitational constant G(t) is zero at the initial singularity and it is increasing with time. The cosmological constant Λ (t) $\propto 1/t^2$ follows from the model of [4, 5, 7, 16] .This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. Finally, the solutions presented in the paper are new and useful for a better understanding of the evolution of the universe in $R - W$ space-time with variable G and Λ.

5. Conclusion

In this paper, we have generalized the solution that an equation of state for perfect fluid is spatially homogeneous and isotropic R-W line element with variable gravitational constant G (t) and cosmological constant Λ (t). The field equations have been solved exactly by using a law of variation of scale factor with a variable cosmological term, i.e. a cosmological term that scales as Λ $\propto \mathrm{R}^{-m}$. The exact $R - W$ model has been obtained in sections 3.1. The models represent a shearing, non-rotating and expanding model of the universe with a big-bang start which approaches isotropy at late times. In section-3.1 m=1 for the universe spatial volume V is zero at t=t* where $t^* = -2t_0/\left(\sqrt{\frac{3a}{7}}\right)$ and expansion scalar θ is infinite, which demonstrates that the universe begins developing with zero volume at t= t^*, with an infinite rate of expansion. Thus the expansion rate increases as time increases.

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